# Performance Analysis of Beamforming for Radar and Communications Coexisting Systems

Jungeun Lee, Hyeonho Noh, and Hyun Jong Yang
School of Electrical and Computer Engineering
Ulsan National Institute of Science and Technology
Ulsan, South Korea
{jungeunlee88, nhh0623, hjyang}@unist.ac.kr

***Abstract*—Several studies have been conducted for designing beam synthesis in radar and communications combined systems, in pursuit of enhancing the performance of the target detection and wireless communications simultaneously. However, their channel models, assuming no characteristics of directionality, are unrealistic in the sense that the radar channels should in general be line-of-sight (LOS), whereas the communications channels are often none-LOS (NLOS). In this paper, the performance of the previous schemes is re-evaluated under a practical environment in which the radar channel is LOS while the communications channel is NLOS. With careful consideration of the directionality of ray-based channels, numerical results reveals clear trade-off between the minimum signal-to-noise-plus-interference-ratio and peak to side lobe ratio.**

***Index Terms*—Radar, communications, line-of-sight (LOS), non-line-of-sight (NLOS), beam synthesis**

## I. Introduction

Recent studies have shown that sharing spectrum for communications and radar systems can significantly increase spectral efficiency [1]–[3]. In multi-antenna systems, proper beam synthesis is crucial to compromise between the communication data rate and the target detection performance. In the viewpoint of antenna configuration, two separated groups of antennas can be used for communications and radar, respectively. On the other hand, all the antennas can be used both for communications and radar, i.e., shared antenna. In this paper, we restrict ourselves to the separated antennas scenario, in which two different beam patterns can be individually and flexibly optimized for communications and target detection functionality.

In the literature, the channels to communications users are assumed to simply follow the complex Gaussian distribution, i.e., non-line-of-sight (NLOS) fading with rich scattering, without careful consideration of feasibility of the scenario. The channel to the radar target must be line-of-sight (NLOS) to apply conventional target detection theory. Thus, the geometric scenario is essential to ascertain the feasibility of the coexistence of NLOS and LOS channels.

In Fig. 1, we first present a feasible scenario under which the radar target is seen by the base station with LOS and where the communications users are surrounded by scatters, thereby yielding NLOS channels. As a result, angle of departure (AoD) becomes critical when considering the channels to the communications users. Previous results on

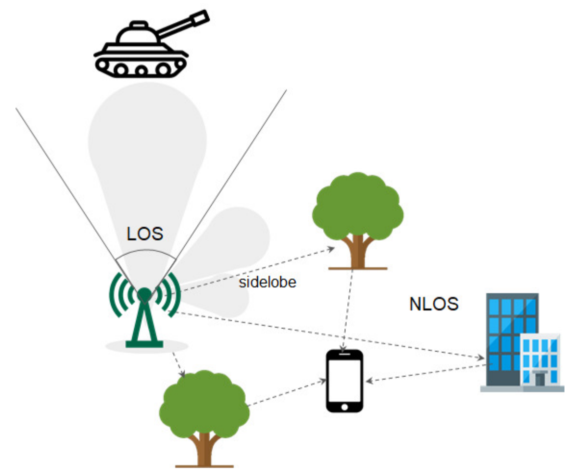


Fig. 1. Feasible system model for the communication and radar combined system

mmWave channel modeling have been borrowed to establish accurate assumptions. We then reformulate the beam synthesis algorithms originally proposed in [1]. Numerical simulations are provided under the considered feasible scenario, which show beampatterns using channel model which have angular information about users' location and the complex Gaussian channel model, respectively.

## II. System Model

Two groups of transmit antennas at the base station, composed of $N_R$ and $N_C$ uniform arrays, respectively, are used for radar and communications, respectively. The received signal of the $i$-th user is expressed as

$$x_i = \mathbf{g}_i^T \sum_{k=1}^{K} \mathbf{w}_k c_k + \mathbf{f}_i^T \mathbf{r} + w_i, \tag{1}$$

where $K$ is the number of users, $\mathbf{g}_i \in \mathbb{C}^{N_C \times 1}$ and $\mathbf{f}_i \in \mathbb{C}^{N_R \times 1}$ are the channel vectors from communication antennas and radar antennas to the $i$-th user. The transmit signal vector of the radar antennas is denoted by $\mathbf{r} \in \mathbb{C}^{N_R \times 1}$, and the communication transmit symbol is denoted by $c_k$. Due to rich scattering around the user, the communication channels are assumed to be NLOS, thus, $w_i$ is the received noise of the

$i$-th user and it follows the complex Gaussian distribution, $CN(0, N_0)$.

The array steering vectors from the $N_R$ radar arrays and from the $N_C$ communications arrays to the target located at the angle $\theta$ are expressed as

$$\mathbf{a}_1(\theta) = [1, e^{j2\pi\Delta\sin(\theta)}, ..., e^{j2\pi(N_R-1)\Delta\sin(\theta)}]^T \in \mathbb{C}^{N_R\times 1}, \quad (2)$$

$$\mathbf{a}_2(\theta) = [1, e^{j2\pi\Delta\sin(\theta)}, ..., e^{j2\pi(N_C-1)\Delta\sin(\theta)}]^T \in \mathbb{C}^{N_C\times 1}. \quad (3)$$

## III. Beam Synthesis

For the separated antennas utilization, a beam synthesis algorithm with zero communication interference from the radar arrays, originally proposed in [1], is reviewed for self-completeness. For the signals transmitted from the radar arrays not to interfere the communication signals at the users, the radar transmit signal vector $\mathbf{r}$ should satisfy the following constraint:

$$\mathbb{E}\{\left|\left|\mathbf{f}_i^T\mathbf{r}\right|\right|\} = \mathbf{f_i}^T\mathbb{E}\{\mathbf{r}\mathbf{r}^H\}\mathbf{f}_i^* = \mathrm{tr}(\mathbf{f}_i^*\mathbf{f}_i^T\mathbf{R}) = 0, \forall i. \quad (4)$$

For generating beampattern, we need the covariance matrix of the transmit signal vector of the radar antennas, $\mathbf{R} = E\{\mathbf{r}\mathbf{r}^H\}$.

$$\min_{\alpha,\mathbf{R}} \sum_{m=1}^{M} \left|\alpha P_d(\theta_m) - \mathbf{a}_1(\theta_m)^H\mathbf{R}\mathbf{a}_1(\theta_m)\right|^2 \quad (5a)$$

$$\text{s.t. } \mathrm{diag}(\mathbf{R}) = \frac{P_0\mathbf{1}}{N_R}, \quad (5b)$$

$$\mathbf{R} \succeq 0, \mathbf{R} = \mathbf{R}^H, \alpha \geq 0, \quad (5c)$$

$$\mathrm{tr}(\mathbf{f}_i^*\mathbf{f}_i^T\mathbf{R}) = 0, \forall i. \quad (5d)$$

For ease of the implementation, the problem (5a) can be equivalently written as

$$\min_{\alpha,\mathbf{R}} \mathrm{t} \quad (6)$$

$$\text{s.t. } \sum_{m=1}^{M} \left|\alpha P_d(\theta_m) - \mathbf{a}_1(\theta_m)^H\mathbf{R}\mathbf{a}_1(\theta_m)\right|^2 \leq t, \quad (7)$$

$$\mathrm{diag}(\mathbf{R}) = \frac{P_0\mathbf{1}}{N_R}, \quad (8)$$

$$\mathbf{R} \succeq 0, \mathbf{R} = \mathbf{R}^H, \alpha \geq 0, \quad (9)$$

$$\mathrm{tr}(\mathbf{f}_i^*\mathbf{f}_i^T\mathbf{R}) = 0, \forall i. \quad (10)$$

The feasible set of the problem (6) is convex, while the cost function is a convex (affine) function. Thus, the problem can be readily obtained by semidefinite programming (SDP).

Solving the problem (6) gives us the generated beampattern $\mathbf{R}$ which has zero interference to the users. Now, the aim is to find $\mathbf{w}_k$, the communication beamforming vector. In [1], for given $\mathbf{r}$, it is suggested to render $\mathbf{w}_k$ such that the radar beampattern of $\mathbf{w}_k$ tends to be the same as the radar beampattern of $\mathbf{r}$. On the other hand, a minimum SINR constraint can be employed to guarantee a minimum data rate of the users.

For given $\mathbf{R}$, the optimization problem for $\mathbf{w}_k$ can be formulated by letting $\mathbf{W}_k = \mathbf{w}_k\mathbf{w}_k^H$ as

$$\min_{\sigma,\mathbf{W}_i} \left|\left|\mathrm{diag}(\mathbf{A}_2^H\sum_{i=1}^{K}\mathbf{W}_i\mathbf{A}_2 - \sigma\mathbf{A}_1^H\mathbf{R}\mathbf{A}_1)\right|\right|^2 \quad (11a)$$

$$s.t. \ \beta_i \geq \Gamma_i, \forall i, \quad (11b)$$

$$P_1 \leq P_c, \quad (11c)$$

$$\sigma \geq 0, \quad (11d)$$

$$\mathbf{W}_i \succeq 0, \mathbf{W}_i = \mathbf{W}_i^H, \quad (11e)$$

where $\{\theta_1, \ldots, \theta_M\}$ are the set of $M$ discrete angle points, $\mathbf{A}_1 = [\mathbf{a}_1(\theta_1), ..., \mathbf{a}_1(\theta_M)] \in \mathbb{C}^{N_R\times M}$ , $\mathbf{A}_2 = [\mathbf{a}_2(\theta_1), ..., \mathbf{a}_2(\theta_M)] \in \mathbb{C}^{N_C\times M}$, and $\Gamma_i$ is the SINR threshold of $i$-th user. The communication transmit power, $P_1$, can be expressed as

$$P_1 = \sum_{k=1}^{K}||\mathbf{w}_k||^2 = \sum_{k=1}^{K}\mathrm{tr}(\mathbf{W}_k). \quad (12)$$

The received SINR, $\beta_i$, can be expressed as

$$\beta_i = \frac{\left|\mathbf{g}_i^T\mathbf{w}_i\right|^2}{\sum_{k=1,k\neq i}^{K}\left|\mathbf{g}_i^T\mathbf{w}_k\right|^2 + \mathbf{f}_i^T\mathbf{R}\mathbf{f}_i^* + N_0} \quad (13a)$$

$$= \frac{\mathrm{tr}(\mathbf{g}_i^*\mathbf{g}_i^T\mathbf{W}_i)}{\mathrm{tr}(\mathbf{g}_i^*\mathbf{g}_i^T\sum_{k=1,k\neq i}^{K}\mathbf{W}_k) + \mathrm{tr}(\mathbf{f}_i^*\mathbf{f}_i^T\mathbf{R}) + N_0}, \quad (13b)$$

where $N_0$ is a variance of the received noise. The problem (11a) is composed of a convex cost function and convex constraints except for (11b). However, using the expression (13b), the constraint (11b) can be converted into a linear constraint, which makes the problem solvable using SDP.

## IV. Channel Models

In [1], the channel vectors $\mathbf{g}_i$ and $\mathbf{f}_i$ are assumed to be independent and identically distributed complex Gaussian without any careful consideration of scenario and channel model. In addition, no directional geometry is considered for the communication users. This assumption cannot be feasible. In fact, though in the NLOS communications environment, $\mathbf{g}_i$ and $\mathbf{f}_i$ should be carefully generated by considering angle of departures of rays. Accurate ray-based channel model can be obtained for $\mathbf{g}_i$ and $\mathbf{f}_i$ based on [4]–[6]. More detailed discussion on the channel model can be found from [7].

## V. Simulation Results

### A. $\Gamma_i$ Design

For simulations, we choose a homogeneous minimum SINR threshold, i.e., $\Gamma_i = \Gamma$, $\forall i$. The goal of the simulation is to investigate the data rate and peak-to-sidelobe-ratio (PSLR). That is, we investigate the trade-off between $\Gamma$ and PSLR. If $\Gamma$ is set too high for given channel conditions so that the problem (11a) has no feasible solution, then we propose to

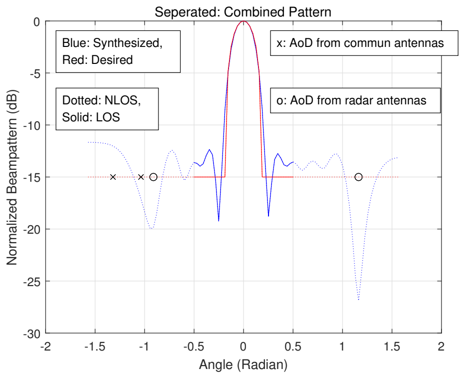


Fig. 2. Beampattern where $\Gamma_i = 5$, $K = 2$, $N_R = 10$, $N_C = 8$, $P_0 = P_1 = 1$, $N_0 = 0.1$, LOS range is $[-\pi/6, \pi/6]$.

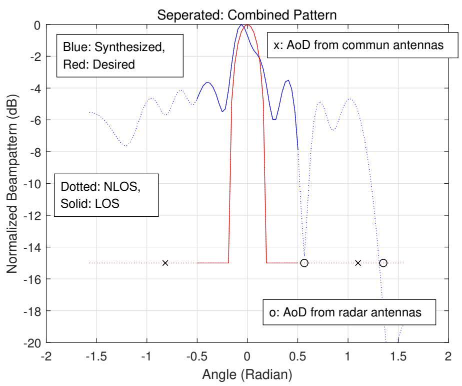


Fig. 3. Beampattern where $\Gamma_i = 80$, $K = 2$, $N_R = 10$, $N_C = 8$, $P_0 = P_1 = 1$, $N_0 = 0.1$, LOS range is $[-\pi/6, \pi/6]$.

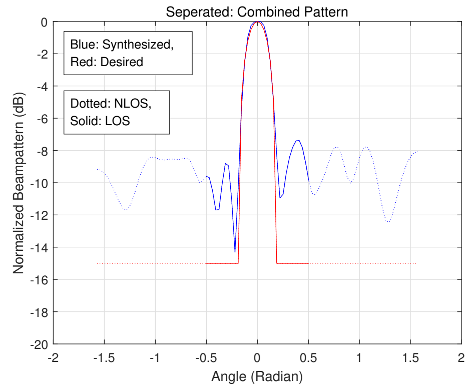


Fig. 4. Beampattern with complex random channels [1] where $\Gamma_i = 5$, $K = 2$, $N_R = 10$, $N_C = 8$, $P_0 = P_1 = 1$, $N_0 = 0.1$, LOS range is $[-\pi/6, \pi/6]$.

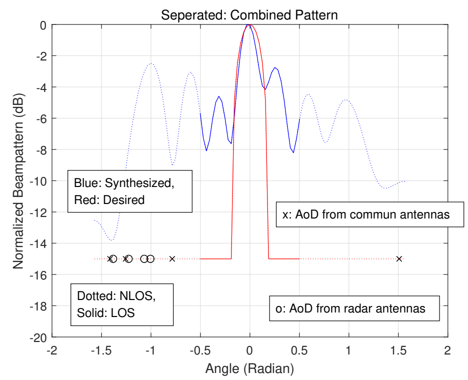


Fig. 5. Beampattern where $\Gamma_i = 5$, $K = 4$, $N_R = 10$, $N_C = 8$, $P_0 = P_1 = 1$, $N_0 = 0.1$, LOS range is $[-\pi/6, \pi/6]$.

obtain $\mathbf{W}_i$ from the maximization problem of the minimum SINR as follows:

$$\max_{\mathbf{W}_i} \quad \min\{\beta_1, \ldots, \beta_K\} \tag{14}$$

$$s.t. \quad P_1 \leq P_c, \tag{15}$$

$$\mathbf{W}_i \succeq 0, \mathbf{W}_i = \mathbf{W}_i^H. \tag{16}$$

### *B. Performance Evaluation*

In Figures 2 and 3 show the beampattern vs. angle for $\Gamma = 5$ and $\Gamma = 40$, respectively, where the other parameters are assumed to be as follows: $N_R = 10$, $N_C = 8$, $P_0 = P_c = 1$, $N_0 = 0.1$, and, $K = 1$. The range of LOS, i.e., possible target presence range, is assumed to be between -$\frac{\pi}{6}$ and $\frac{\pi}{6}$. The channel vector is generated based on the ray-based directional channel model as in [4]–[6]. In the directional channel model, the beampattern due to $\mathbf{R}$, the solution of (6), should have nulls in the angles where the users are located as shown in Fig. 2. However, as $\Gamma$ becomes high, the beampattern due to $\mathbf{W}_i$, becomes significantly different from the desired pattern.

Fig. 4 shows the beampattern vs. angle when the channels are merely assumed to be complex Gaussian as in [1]. The parameters are all the same as in Fig. 2. As seen from the figures, if the channel is assumed to be directional, the PSLR can be much higher than with the assumption of non-directional random channel case.

Fig. 5 shows the beampattern vs. angle where the number of users, $K$, is 4. The rest of the parameters are the same as in Fig. 2. As $K$ increases, the sum data rate increases, and thus the PSLR becomes smaller.

## Acknowledgment

This research was supported by the MSIT(Ministry of Science and ICT), Korea, under the ITRC(Information Technology Research Center) support program(IITP-2018-2017-0-01635) supervised by the IITP(Institute for Information & communications Technology Promotion), the National Research Foundation of Korea(NRF) grant funded by the Korea government(MOE) (NRF-2016R1D1A1B03935685), IITP grant funded by the Korea government(MSIT) (No.2018-0-00958, Development of joint Electrical/Mechanical Drone Beamforming based on Target Detection and Precise Attitude Control)

## VI. Conclusion

We have reevaluated the performance of beamsynthesis for communications and radar combined systems under the feasible scenario considering the directional location of the users, where separate antennas are used for the transmission of communication and radar signals. It turend out that the beam synthesis performance is lower than expected in the previous studies. In our future work, we will redesign the beam synthesis algorithm to take into account the impact of the directional location of the users.

## References


[1] F. Liu, C. Masouros, A. Li, H. Sun, and L. Hanzo, "MU-MIMO communications with MIMO radar: From co-existence to joint transmission," *IEEE Transactions on Wireless Communications*, vol. 17, no. 4, pp. 2755–2770, April 2018.

[2] L. Zheng, M. Lops, X. Wang, and E. Grossi, "Joint design of overlaid communication systems and pulsed radars," *IEEE Transactions on Signal Processing*, vol. 66, no. 1, pp. 139–154, Jan 2018.

[3] W. W. Christian Sturm, Thomas Zwick, "An OFDM concept for joint radar and communications operations," *IEEE 69th Vechicular Technology Conference*, pp. 1–5, 2009.

[4] M. R. Akdeniz, Y. Liu, M. K. Samimi, S. Sun, S. Rangan, T. S. Rappaport, and E. Erkip, "Millimeter wave channel modeling and cellular capacity evaluation," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1164–1179, June 2014.

[5] 3GPP, "3gpp tr 25.996," 3rd Generation Partnership Project (3GPP), 3GPP, Valbonne, France, Tech. Rep. TR 25.996 v12.1.0, TR 25.996, Dec. 2009. [Online]. Available: http://www.qtc.jp/3GPP/Specs/25996-900.pdf

[6] M. K. Samimi and T. S. Rappaport, "Ultra-wideband statistical channel model for non line of sight millimeter-wave urban channels," in *2014 IEEE Global Communications Conference*, Dec 2014, pp. 3483–3489.

[7] H. Noh, J. Lee, and H. Yang, "Beam synthesis under feasible scenarios for radar and communications combined systems," in *Proc. International Conference on ICT Convergence (ICTC)*, submitted.